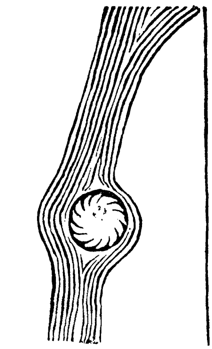

Prof. J. E. Oliver, Cornell University, Ithaca, N. Y. writes:

Your correspondent's "query" is answered by some standard writer on Physics (probably Dagain or Ganot, or both,)—and the picture there is better than I can draw.

The stream, adhering to the ball, is deflected toward it. If the ball is to the right of the stream's axis more water passes to the ball's left and is deflected to the right than the reverse; $\therefore$ the stream is, on the whole, deflected to the right, $\therefore$ the ball is drawn to the left, since action and reaction are opposite.

Note on the Ellipse.—From the well known property of the ellipse, that the rectangle of the abscissas is to the square of the ordinate as the square of the major axis is to the square of the minor axis, we get the equation

$$y = \frac{b}{a} \sqrt{a^2 - x^2},$$

in which a and b are the semi-axes and x and y the co-ordinates; the origin being at the center. If we put

$$x = \frac{2\,a\,n}{n^2+1} \text{ or } \frac{a(n^2-1)}{n^2+1},$$

y will always be rational and equal to

$$b\,.\,\frac{n^2-1}{n^2+1} \text{ or } b\,.\,\frac{2\,n}{n^2+1}.$$

Assume $n^2 + 1 = a$, then will $2n$ and $n^2 - 1$ represent the abscissas corresponding to the ordinates

$$b\,.\,\frac{n^2-1}{n^2+1} \text{ and } b\,.\,\frac{2\,n}{n^2+1},$$

where n may be any number whatever greater than *one.*

Hence, if we divide the semi-major axis of any ellipse into $n^2 + 1$ equal parts we may find two points in the axis whose distance from the center or from the extremity of the axis may be expressed in integral parts of n, and each of which has a rational ordinate.

Example.—Put $n = 2$, then is the semi-major axis $= n^2 + 1 = 5$, and the two points in the axis are respectively $2\,n$ and $n^2 - 1$, or 4 and 3 dis-

tant from the center, or 1 and 2 distant from the extremity of the axis, and their corresponding ordinates are $\frac{3}{5}b$ and $\frac{4}{5}b$.

If $n = 3$, we have $n^2 + 1 = 10$, and the distances of the two points from the extremity of the axis are 4 and 2, and their corresponding ordinates are $\frac{4}{5}b$ and $\frac{3}{5}b$. If $n = 4$, $n^2 + 1 = 17$, and the distances of the two points from the extremity of the axis are 9 and 2, and their corresponding ordinates are $\frac{15}{17}b$ and $\frac{8}{17}b$. By assigning other values to n other rational ordinates may be found to any extent that may be desired.

It is remarkable that whatever value may be assigned to n, one of the points in the axis that has a rational ordinate will always be at a distance from the extremity of the axis equal to two of the units in n.

Problem.—To integrate $\dfrac{d\,x}{1 + e\cos x}$.

$$\text{We have } \frac{d\,x}{1 + e\cos x} = \frac{d\,x}{1 + e\cos^2 \frac{1}{2}x - e\sin^2 \frac{1}{2}x}.$$

Put tan. $\frac{1}{2}x = z$; then $\frac{1}{2}\sec^2 \frac{1}{2}x\,d\,x = d\,z$, and $\sec^2 \frac{1}{2}x = 2\,d\,z$.
Therefore the required integral

$$= \int \frac{\dfrac{d\,x}{\cos^2 \frac{1}{2}x}}{\dfrac{1}{\cos^2 \frac{1}{2}x} + e - e\tan^2 \frac{1}{2}x} = \int \frac{\sec^2 \frac{1}{2}x\,d\,x}{\sec^2 \frac{1}{2}x + e - e\tan^2 \frac{1}{2}x}$$

$$= \int \frac{2\,d\,z}{1 + z^2 + e - e\,z^2} = \frac{2}{1+e}\int \frac{d\,z}{1 + \dfrac{1-e}{1+e}z^2}$$

$$= \frac{2}{1+e}\sqrt{\frac{1+e}{1-e}}\int \frac{\sqrt{\dfrac{1-e}{1+e}}\,d\,z}{1 + \dfrac{1-e}{1+e}z^2}$$

$$= \frac{2}{\sqrt{1-e^2}}\tan^{-1}\sqrt{\frac{1-e}{1+e}}\,z = \frac{2}{\sqrt{1-e^2}}\tan^{-1}\sqrt{\frac{1-e}{1+e}}\tan \tfrac{1}{2}x + C.$$

—James E. Clark, Professor of Mathematics, *William Jewell College*, Liberty, Mo.